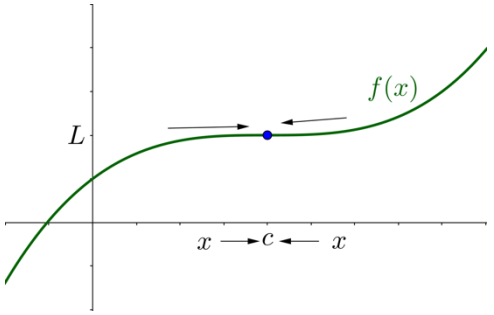
Precalculus

12-01 Introduction to Limits

Limit

If $f(x)$ becomes _____ close to a unique number L as x _____ c from either side, then the limit of $f(x)$ as x approaches c is _____.

$$\lim_{x \to c} f(x) = L$$



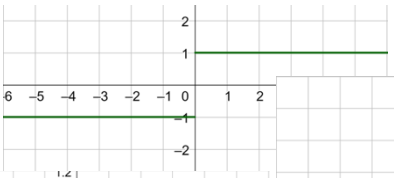
Ways to find limits

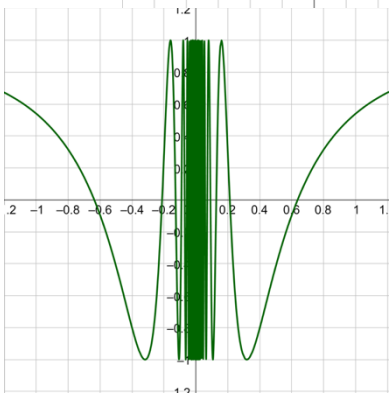
- Estimate Numerically (_____)

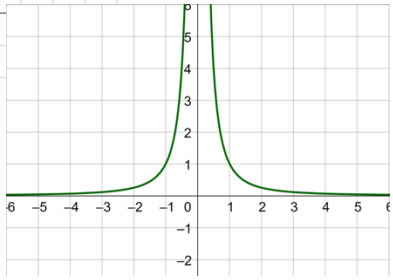
$$\lim_{x \to -2} \frac{x^2 + 4x + 4}{x + 2}$$

Limits that fail to exist

1. $f(x)$ approaches _____ numbers from both sides
2. $f(x)$ increases or decreases without _____
3. $f(x)$ _____ between 2 fixed values







Properties of Limits

- $\lim_{x \to c} b = b$
- $\lim_{x \to c} x = c$
- $\lim_{x \to c} x^n = c^n$
- $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$
- Let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$
  - $\lim_{x \to c} bf(x) = bL$
  - $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
  - $\lim_{x \to c} f(x)g(x) = LK$
  - $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$
  - $\lim_{x \to c} [f(x)]^n = L^n$

Evaluate

$\lim_{x \to 2} 3x^2$

$\lim_{x \to 1} (4x^3 - 2x^2 + 17)$

$\lim_{x \to 2} \frac{x^2 - 4}{x}$

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12-02 Evaluating Limits

Indeterminant Form

$$\lim_{x\to c} f(x) = \frac{0}{0}$$

Dividing out technique

1. _____
2. _____ common factors
3. Then find the _____

Evaluate $\lim_{x\to 3} \frac{x^2-8x+15}{x-3}$

Rationalizing Technique

- Get _____ out of _____
- _____ by _____ of _____

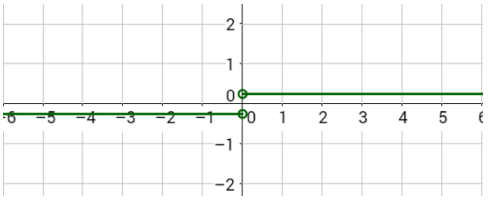
Evaluate $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$

One-sided Limits

- Limit found form only _____ direction
- $\lim_{x\to c^-} f(x)$ - from _____
- $\lim_{x\to c^+} f(x)$ - from _____

Evaluate

$\lim_{x\to 0^-} \frac{|x|}{4x}$



$\lim_{x\to 0^+} \frac{|x|}{4x}$

A limit from calculus

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- _____ gives indeterminant case

For the function $f(x) = 2x^2 + 1$ find $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

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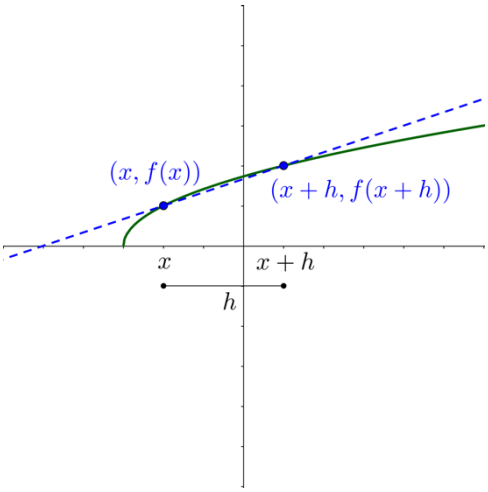
12-03 Derivatives



Calculus is based on two main problems

- Finding the _____ of the tangent line to a function (finding _____)
- Find _____

$$Slope\ of\ tangent\ line = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivative

$$\boldsymbol{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

Gives a function for slope, or rate of change, of a function

Find the slope of $f(x) = x^3$ at $(2,\ 8)$

Find the derivative of $f(x) = x^2 - 2$

Find the derivative of $f(x) = \sqrt{x} + 1$

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12-04 Limits at Infinity and Limits of Sequences

Limits at Infinity

$$\lim_{x\to\infty}\frac{1}{x^r}=0$$

$$\lim_{x\to-\infty}\frac{1}{x^r}=0$$

Evaluate $\lim_{x\to\infty}\frac{1+5x-3x^3}{x^3}$

Shortcut

- N = degree of _____
- D = degree of _____
- N < D → _____
- N = D → _____
- N > D → _____

Evaluate

$\lim_{x\to\infty}\frac{-x+4}{5x^2+2}$

$\lim_{x\to\infty}\frac{-x^2+4}{5x^2+2}$

Limits of Sequences

- If terms of a sequence approach a _____ as $n\to\infty$, then it _____.
- Otherwise, it _____.

Find the limit of the sequence $a_n=\frac{(n-3)(4n-1)}{4-3n-n^2}$

Find the limit of $a_n=\frac{5}{n^3}\cdot\left[\frac{n(n+1)(2n+1)}{6}\right]$.

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12-05 Integrals

Properties of Sums

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4+2n^3+n^2}{4}$$

- Associative Property

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

- Distributive Property (Factoring)

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

Find the limit of $S_n = \sum_{i=1}^{n} \frac{i-5}{n^2}$ as $n \to \infty$

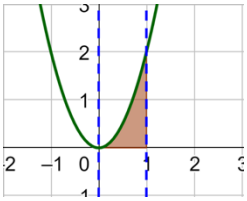
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i-5}{n^2}$$

The Area Problem

- Find the area between the graph and the *x*-axis between two *x*-values *a* and *b*

$$Area = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right)\left(\frac{b-a}{n}\right)$$

Find the area bounded by $f(x) = 2x^2$ and $x = 0$ and $x = 1$



Find the area bounded by $f(x) = 4x - x^2$ and $x = 1$ to $x = 3$