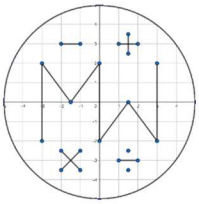


Mr. Wright's Math Extravaganza

Precalculus

Introduction to Calculus

Level 2.0: 70% on test, Level 3.0: 80% on test, Level 4.0: level 3.0 and success on applications

Score	I Can Statements
4.0	☐ I can demonstrate in-depth inferences and applications that go beyond what was taught.
3.5	In addition to score 3.0 performance, partial success at score 4.0 content
3.0	☐ I can evaluate limits. ☐ I can find the derivative of a function. ☐ I can evaluate an integral.
2.5	No major errors or omissions regarding score 2.0 content, and partial success at score 3.0 content
2.0	☐ I can evaluate limits using a table. ☐ I can evaluate limits using substitution. ☐ I can evaluate indeterminant limits. ☐ I can find the slope of a tangent line to a function. ☐ I can evaluate limits at infinity. ☐ I can evaluate the limit of a sequence. ☐ I can find the limit of sums as n approaches infinity.
1.5	Partial success at score 2.0 content, and major errors or omissions regarding score 3.0 content.
1.0	With help, partial success at score 2.0 content and score 3.0 content.
0.5	With help, partial success at score 2.0 content but not at score 3.0 content.
0.0	Even with help, no success.

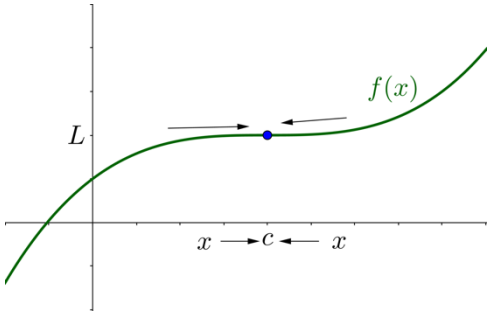
Precalculus

12-01 Introduction to Limits

Limit

If $f(x)$ becomes _____ close to a unique number L as x _____ c from either side, then the limit of $f(x)$ as x approaches c is _____.

$$\lim_{x \to c} f(x) = L$$



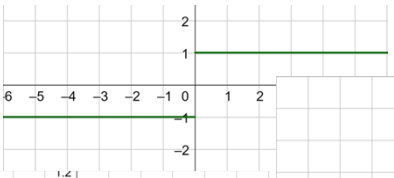
Ways to find limits

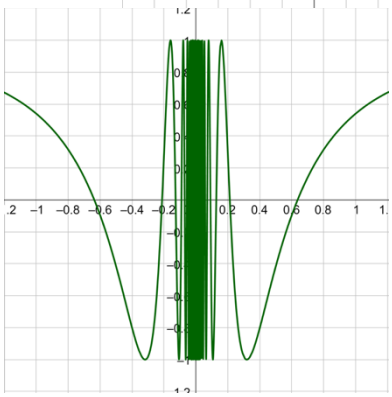
- Estimate Numerically (_____)

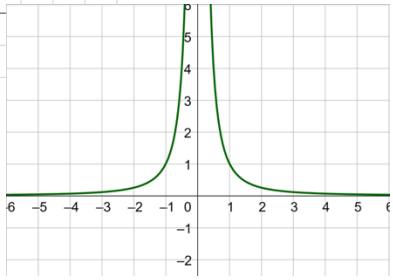
$$\lim_{x \to -2} \frac{x^2 + 4x + 4}{x + 2}$$

Limits that fail to exist

1. $f(x)$ approaches _____ numbers from both sides
2. $f(x)$ increases or decreases without _____
3. $f(x)$ _____ between 2 fixed values







Properties of Limits

- $\lim_{x \to c} b = b$
- $\lim_{x \to c} x = c$
- $\lim_{x \to c} x^n = c^n$
- $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$
- Let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$
  - $\lim_{x \to c} bf(x) = bL$
  - $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
  - $\lim_{x \to c} f(x)g(x) = LK$
  - $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$
  - $\lim_{x \to c} [f(x)]^n = L^n$

Evaluate

$\lim_{x \to 2} 3x^2$

$\lim_{x \to 1} (4x^3 - 2x^2 + 17)$

$\lim_{x \to 2} \frac{x^2 - 4}{x}$

Precalculus

12-02 Evaluating Limits

Indeterminant Form

$$\lim_{x\to c} f(x) = \frac{0}{0}$$

Dividing out technique

1. _____
2. _____ common factors
3. Then find the _____

Evaluate $\lim_{x\to 3} \frac{x^2-8x+15}{x-3}$

Rationalizing Technique

- Get _____ out of _____
- _____ by _____ of _____

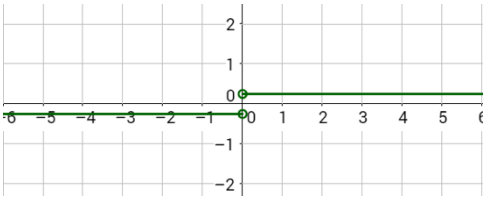
Evaluate $\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$

One-sided Limits

- Limit found from only _____ direction
- $\lim_{x\to c^-} f(x)$ - from _____
- $\lim_{x\to c^+} f(x)$ - from _____

Evaluate

$\lim_{x\to 0^-} \frac{|x|}{4x}$



$\lim_{x\to 0^+} \frac{|x|}{4x}$

A limit from calculus

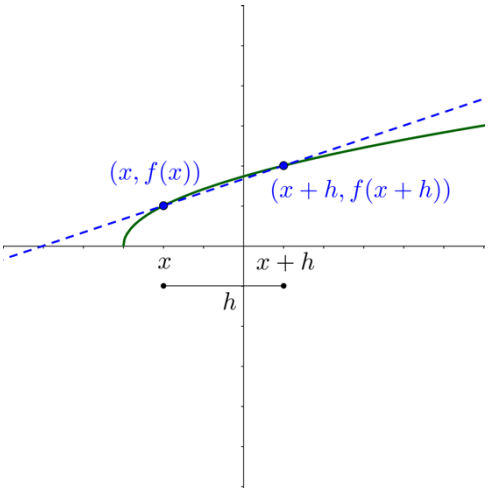
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

- _____ gives indeterminant case

For the function $f(x) = 2x^2 + 1$ find $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$

Precalculus

12-03 Derivatives



Calculus is based on two main problems

- Finding the _____ of the tangent line to a function (finding _____)
- Find _____

$$Slope\ of\ tangent\ line = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivative

$$\boldsymbol{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

Gives a function for slope, or rate of change, of a function

Find the slope of $f(x) = x^3$ at $(2,\ 8)$

Find the derivative of $f(x) = x^2 - 2$

Find the derivative of $f(x) = \sqrt{x} + 1$

Precalculus

12-04 Limits at Infinity and Limits of Sequences

Limits at Infinity

$$\lim_{x\to\infty}\frac{1}{x^r}=0$$

$$\lim_{x\to-\infty}\frac{1}{x^r}=0$$

Evaluate $\lim_{x\to\infty}\frac{1+5x-3x^3}{x^3}$

Shortcut

- N = degree of _____
- D = degree of _____
- N < D → _____
- N = D → _____
- N > D → _____

Evaluate

$\lim_{x\to\infty}\frac{-x+4}{5x^2+2}$

$\lim_{x\to\infty}\frac{-x^2+4}{5x^2+2}$

Limits of Sequences

- If terms of a sequence approach a _____ as $n\to\infty$, then it _____.
- Otherwise, it _____.

Find the limit of the sequence $a_n=\frac{(n-3)(4n-1)}{4-3n-n^2}$

Find the limit of $a_n=\frac{5}{n^3}\cdot\left[\frac{n(n+1)(2n+1)}{6}\right]$.

Precalculus

12-05 Integrals

Properties of Sums

$$\sum_{i=1}^{n} c = cn$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4+2n^3+n^2}{4}$$

- Associative Property

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

- Distributive Property (Factoring)

$$\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

Find the limit of $S_n = \sum_{i=1}^{n} \frac{i-5}{n^2}$ as $n \to \infty$

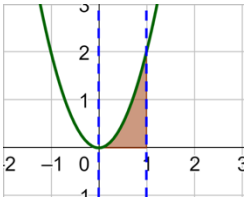
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i-5}{n^2}$$

The Area Problem

- Find the area between the graph and the *x*-axis between two *x*-values *a* and *b*

$$Area = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(a + \frac{b-a}{n} i\right)\left(\frac{b-a}{n}\right)$$

Find the area bounded by $f(x) = 2x^2$ and $x = 0$ and $x = 1$



Find the area bounded by $f(x) = 4x - x^2$ and $x = 1$ to $x = 3$